a-g Integrated Math 3

Integrated Math 3 is the third course of a three year integrated high school sequence that combines topics in Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. The study of Number and Quantity will introduce students to the complex number system in the context of polynomial identities and equations. The study of Algebra will extend the students' knowledge of basic algebra to include polynomials and rational expressions and equations. The study of Functions will expand students' understanding of functions to include polynomial, rational and logarithmic, radical, and trigonometric functions. The study of Geometry will extend the prior knowledge of trigonometry to general triangles and connect algebra and geometry in modeling situations. The study of Probability and Statistics will focus on applying a variety of methods to draw inferences and make conclusions from data.

Each unit contains exercises and activities, including performance tasks, that require students to communicate mathematically by showing their series of calculations and/or explaining their reasoning in solving problems. Most units contain exploration activities that allow students to develop deeper mathematical understanding of a concept as they are guided through a process of investigation.

Student understanding will be assessed throughout each semester with unit tests and at the end of each semester with a comprehensive semester final exam.

Two specific textbooks can be used with this course: *Integrated Mathematics 3*, 2015 edition by Houghton Mifflin Harcourt, or *Mathematics III Common Core*, 2014 edition by Pearson Education. However, the course is written so that it can be used with any textbook aligned to the CA Common Core State Standards for Mathematics.

Course Content:

Geometric Measurement and Modeling

In this unit, students will review and extend their knowledge of measurement of two-dimensional and three-dimensional objects. The unit begins with generating solid figures by rotating two-dimensional figures and visualizing cross-sections of three-dimensional figures. Students will find areas of the cross-sections and other plane figures, and they will find surface areas and volumes of solid figures. Students will then examine and determine the relationships between perimeters, areas, and volumes of similar figures. Students will also solve real-world problems involving geometric measurement, such as density and design. The modeling problems will involve visualizing rotations of plane figures, and/or applying perimeter, area, or volume in a real-world context. The problems may also draw upon students' prior knowledge of special triangles, right triangle trigonometry, and dilations on the coordinate plane.

Unit Assignment(s):

Sample Assignment: **Modeling and Density**

Students will be given quantitative information and be asked to calculate and compare the densities or population densities and explain the meaning of the calculation in the context of the problem. Here are some examples of the tasks:

1) Given the dimensions of a trapezoid-shaped country and its population, calculate the population density in persons per square mile.

2) Given the dimensions of a fish tank and the number of fish in the tank, calculate the population density and assign appropriate units.

3) Given the mass and volume of several metals, calculate the densities of each substance, assign appropriate units, and rate the metals from most to least dense.

4) Given the mass and the dimensions (radius and length) of several tree logs, calculate the densities of each log, assign appropriate units, and rate the logs from most dense to least dense.

5) Given the radius of a spherical-shaped gas tank and the total number of BTUs (British thermal units) in the tank, calculate the number of BTUs per unit of volume and interpret the meaning of this measurement.

Students complete the tasks, showing all of their calculations to justify their answers. From this assignment, students learn that the density of a substance is a ratio of mass to volume, and population density is a ration of

population to area.

Polynomial Functions, Expressions, and Equations

In this unit, students will extend their knowledge of linear and quadratic equations to higher degree polynomial equations. They will explore transformations of the graph of a function--translation, reflection, stretch, and compression--and generalize transformation rules that can be used with any type of function. Students learn how to find the inverse of a function and demonstrate understanding that the inputs and outputs of the original function are interchanged in the inverse function. Students graph polynomial functions and identify key features, such as zeros, turning points, and end behavior. Students will add, subtract, multiply, and divide polynomial expressions, and apply these operations to real-world problems. Students will learn the binomial theorem to expand powers of binomials. Binomial probability is also introduced as an application of the binomial theorem.

Students will use their knowledge of polynomial graphs and expressions to solve polynomial equations. Students will learn to factor polynomials and use those factors to find the zeros of a function and the solutions to a polynomial equation. Students will extend their knowledge of factoring quadratics to factoring higher degree polynomials using long division and synthetic division. Students will use the Rational Root Theorem to find possible zeros and then test these zeros to find factors. Students will learn the Fundamental Theorem of Algebra and be able to find all of the complex roots of a polynomial function.

Unit Assignment(s):

Sample Assignment: Solving Polynomial Equations

Given a fourth or fifth degree polynomial equation, students will work individually or in pairs, to answer to complete the following series of tasks:

1) Determine how many solutions and the possible of combinations of types of roots (real or non-real and their multiplicities) for each equation.

2) Graph the related function, and use the graph to determine which combination from task 1 is the correct combination of types of zeros.

3) Find the zeros of the function algebraically by re-writing the polynomial as a product of linear polynomials and/or non-factorable quadratic polynomials, then set each factor equal to zero and solve for the variable to determine the zeros.

4) Compare the results of tasks 2 and 3.

Students must synthesize several concepts in order to complete this task. They must understand the Fundamental Theorem of Algebra to determine the number of zeros, and understand the end-behavior of the graphs of polynomial functions along with the Complex Conjugate Root Theorem, to determine the possible combinations of types of roots. To find the roots algebraically, students will potentially utilize the Rational Root Theorem, factoring techniques, the quadratic formula, and synthetic division.

Rational Functions, Expressions, and Equations

In this unit, students will build on what they have learned about polynomials and apply that knowledge to rational functions, expressions, and equations. They will learn about the characteristics of the graphs of simple rational functions and then transform those graphs, and create tables of values, to graph more complicated functions. Students will also extend prior knowledge of operations with rational numbers, along with factoring polynomials, to operations with rational expressions. Students will find a common denominator when adding and subtracting rational expressions, and cancel common factors in the numerator and denominator (multiplying by 1) when multiplying rational expressions. Lastly, students will learn to solve rational equations both graphically and algebraically. All of these concepts will be applied to real-world problems.

Unit Assignment(s):

Sample Assignment: Rational Function Matching Activity

Students will be given eight transformed parent rational functions, f(x) = 1/x, and eight corresponding graphs. Students match each function with the correct graph applying what they have learned about the effect on the function of translating, stretching and compressing the graph. Students will also need a firm grasp of the key features of the graph of f(x)=1/x to complete this activity. This includes the asymptotes of the graph, and some of the key points on the graph.

Radical Functions, Expressions, and Equations

Similar to the units on polynomials and rational functions, this unit will revisit some of the same concepts--graphing, simplifying expressions, and solving equations--but now involving radicals. Students will first learn the connection between quadratic and square root functions and between cubic and cube root functions. Students will learn the characteristics of the parent square root and cube root functions and then transform these graphs when given more complicated radical functions. Students will use the graphs to model real-world situations involving square root or cube root functions. Students will then extend their knowledge of the rules of exponents to include rational exponents. They will practice simplifying expressions that involve rational exponents, radicals, or both. Lastly, students will extend their knowledge of solving one-variable equations to equations that contain radicals. They will learn to raise both sides of an equation to the same power to eliminate the radical, and then use basic algebra techniques to isolate the variable.

Unit Assignment(s):

Sample Assignment: Simplifying Rational-Exponent Expressions

Given several complex expressions containing rational exponents, students simplify the expression and write a justification for each step. Students learn that the properties of rational exponents are the same as the properties of integer exponents. They learn to apply these properties one at a time to simplify a complex expression into a series of equivalent expressions. They also learn that there is often more than one sequence of steps that will lead to a simplified expression.

Exponential and Logarithmic Functions and Equations

The unit begins with a study of arithmetic and geometric sequences and series. Students make the connection between arithmetic sequences and linear functions and between geometric sequences and exponential functions. Students explore the key features of exponential growth and decay functions and apply them to real-world problems, including those with base *e*. Inverse functions are then revisited to introduce the concept of logarithmic functions. Students learn the features of the graph of the logarithmic function, and apply the graph to real-world situations. Students learn and apply the properties of logarithms to rewrite logarithmic expressions and solve logarithmic and exponential equations.

In this unit, students synthesize what they have learned up to this point about functions, focusing on linear, quadratic, and exponential functions, and choose an appropriate model for a given situation or set of data.

Unit Assignment(s):

Sample Assignment: Linear, Quadratic or Exponential Regression

Given a set of data in table form, students will create a scatter plot and determine the best function (linear, quadratic, or exponential) to model the data. Students will perform a regression on a graphing utility, record the model and the definitions of the variables, graph the model along with the data on the graphing utility, and then assess how well the model fits the data. Students learn to compare these three functions and analyze the appropriateness of a model for the given data. They also make decisions about the domain for which the model is a good fit.

Trigonometric Functions

This unit begins with an extension of the students' knowledge of right triangle trigonometry. Problem solving using right triangles is reviewed and then extended, using the Law of Sines and Law of Cosines to solve general triangles. The understanding of trigonometric ratios is extended to the unit circle, where students learn radian measure of angles and the trigonometric functions of any real number. Students learn to graph the six trigonometric functions. Students learn to graph the parent functions for Sine, Cosine, and Tangent, and then extend what they have previously learned about transforming graphs to include translating, stretching, compressing, and reflecting the Sine, Cosine, and Tangent graphs. Students then learn to graph the reciprocal functions and apply the graphs to

real-world situations.

Unit Assignment(s):

Sample Assignment: Transforming the Sine and Cosine Graphs

Students will be given sine or cosine functions in the form $f(x) = a\sin bx + c$ or $f(x) = a\cos bx + c$. For each function, students will complete a chart indicating the amplitude, period, and midline of the graph. Then students will match each function with its corresponding graph. After the matching activity has been completed, students check their answers by graphing each function on a graphing calculator. In this activity, students demonstrate understanding of the affects of *a, b* and *c* on the amplitude, period, and midline, of the graph of the parent functions.

Statistics and Decision Making

In this unit, students examine both quantitative and qualitative data to make inferences about a population and make decisions based on those inferences. Students learn that standard deviation is a measure of how far the numbers in a data set deviate from the mean. They will learn to calculate the standard deviation of small data sets by hand using an algebraic formula and of larger data sets using technology. Students will use the mean and standard deviation to fit data to a normal distribution and make inferences about the population. Students will also recognize that not all data sets fit a normal distribution.

Students explore the importance of randomness when sampling a population. They will consider various scenarios using sample surveys, experiments, and observational studies, and then determine what conclusions can be made about the population, if any, and justify their conclusions.

Unit Assignment(s):

Sample Assignment: Normal Distribution

Students will explore the relationship between real world data and the normal distribution, by collecting, graphing, and analyzing data. Students will choose a data collection topic, gather the data from at least 20 different sources, and then create a normal distribution graph, after calculating the mean and standard deviation of the data set. Students will then answer questions related to z-scores, range of data values corresponding to a certain percentage within the mean, and discuss the generalizations that can be inferred from the sample. Students will learn to formulate a topic and accurately gather quantitative data. They will practice fitting data they have collected to a normal distribution and consider the appropriateness of the normal distribution.

Course Materials:

Textbook Title:

CA Standards-based textbook (This course is aligned to two textbooks: Integrated Mathematics 3, 2015 edition by Houghton Mifflin Harcourt, or Mathematics III Common Core, 2014 edition by Pearson Education. However, the course is written so that it can be used with any textbook aligned to the CA Common Core State Standards for Mathematics.)